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Estimation of Transition Probabilities of Credit Ratings for Several Companies

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Abstract. This paper attempts to estimate the transition probabilities of credit ratings for a number of companies whose ratings have a dependence structure. Binary codes are used to represent the index of a company together with its ratings in the present and next quarters. We initially fit the data on the vector of binary codes with a multivariate power-normal distribution. We next compute the multivariate conditional distribution for the binary codes of rating in the next quarter when the index of the company and binary codes of the company in the present quarter are given. From the conditional distribution, we compute the transition probabilities of the company's credit ratings in two consecutive quarters. The resulting transition probabilities tally fairly well with the maximum likelihood estimates for the time-independent transition probabilities.

INTRODUCTION

Credit risk is the risk of loss due to the probability that an obligor is unable or unwilling to pay its credit. In the New Basel Capital Accord (Basel II), the matrix of transition probabilities between rating classes plays an essential role in the supervision of banking activities.

Multivariate Markov chain model for credit ratings has been used to estimate credit transition matrices ([1] - [3]). The method based on multivariate Markov chain was applied to the rating data of 15 industries in Taiwan to estimate the probability of the j-th $(1 \leq j \leq 15)$ company's credit rating in the next quarter as a function of the ratings of all the companies in the current quarter [4]. Multivariate power-normal mixture distribution was used to fit the rating data of 10 companies in the electronic industries in Taiwan in order to estimate the transition probability of the future credit rating of a given company in the next quarter as a function of all the 10 companies' credit ratings in the current quarter [5].

In the situation in which the number of companies under investigation is fairly large but the number of quarters spanning the dataset is fairly small, we may not be able to estimate the transition probability of the future credit rating of a given company in the next quarter as a function of all the companies' credit ratings in the current quarter. In this paper, we show that in such situation, we are able to estimate the transition probability of the future credit rating of the given company as a function of only the given company's rating in the present quarter. The procedure involves the use of the following binary codes.

The index of the i-th company in a group of N companies may be represented by the $N-1$ binary codes $0 \cdots 010 \cdots 0$ in which the value "1" takes the i-th position for $1 \leq i \leq N-1$, while the N-th company may be represented by $N-1$ zeros. In the case when the number of possible credit ratings is M, the rating j of a company may be represented by the $M-1$ binary codes $0 \cdots 010 \cdots 0$ in which the value "1" takes the j-th position for $1 \leq j \leq M-1$, while the rating M may be represented by $M-1$ zeros.

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Consider a vector $\mathbf{y}$ of $[N-1+2(M-1)]$ components consisting of the codes for the index of a company together with those for the company's ratings in the present and next quarters. Initially we fit the data for $\mathbf{y}$ with a multivariate power-normal distribution. When the rating of a given company in the present quarter is given, we use the fitted multivariate power-normal distribution to find a conditional distribution for the codes of the company's rating in the next quarter. From the conditional distribution, we compute the transition probability of the company's rating in the next quarter. The resulting transition probabilities are found to agree fairly well with those obtained by the method of maximum likelihood.

The layout of the paper is as follows. In Section 2, we describe the computation of the transition probabilities of credit ratings of M companies. Section 3 presents the numerical results for the transition probabilities obtained from the data on the quarterly credit ratings of 10 companies in 15 years taken from the database of the Taiwan Economic Journal (TEJ). Section 4 concludes the paper.

COMPUTATION OF TRANSITION PROBABILITIES USING MULTIVARIATE POWER-NORMAL DISTRIBUTION

Consider the following vector $\mathbf{y}$ consisting of k correlated random variables:

$$\mathbf{y} = \boldsymbol{\mu} + \mathbf{H}\boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{H}$ is an orthogonal matrix, $\boldsymbol{\mu} = E(\mathbf{y})$, and $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_k$ are uncorrelated random variables. Furthermore, assume that ε_i can be expressed as

$$\varepsilon_i = \sigma_i [\widetilde{\varepsilon}_i - E(\widetilde{\varepsilon}_i)] \Big/ \{\mathrm{var}(\widetilde{\varepsilon}_i)\}^{1/2} \tag{2}$$

where $\sigma_i > 0$ is a constant, and $\widetilde{\varepsilon}_i$ has a power-normal distribution [6] with parameters λ_i^+ and λ_i^- [7].

From the multivariate power-normal distribution for the vector $\mathbf{y}$ of $[N-1+2(M-1)]$ binary codes, we generate a large number N_s of the values of $\mathbf{y}$. The components of $\mathbf{y}$ may be divided into three groups of which group 1 consists of the initial N - 1 components, group 2 consists of the next M – 1 components and group 3 consists of the last M – 1 components. Let $N_1 = N-1$ and $N_2 = N_3 = M-1$. The distance of the N_j components $y_1^{(j)}\ y_2^{(j)} \cdots y_{N_j}^{(j)}$ in Group j of $\mathbf{y}$ from the vector of N_j components $(0, \cdots, 0, 1, 0, \cdots 0)$ with the only component of "1" taking the i-th position is given by

$$D_i^{(j)} = \left\{ \sum_{\substack{i'=1 \\ i' \neq i}}^{N_j} y_{i'}^{(j)2} + \left(y_i^{(j)} - 1 \right)^2 \right\}^{1/2} \tag{3}$$

Let $D_0^{(j)} = \sum_{i'=1}^{N_j} y_{i'}^{(j)2}$. Let $i^{(j)*}$ be the value of i such that among $D_0^{(j)}, D_1^{(j)}, D_2^{(j)}, \cdots,$ and $D_{N_j}^{(j)}$, $D_{i^{(j)*}}^{(j)}$ has the minimum value. Next let

$$i^{(j)} = \begin{cases} i^{(j)*} & , 1 \le i^{(j)*} \le N_j \\ N_j + 1, & i^{(j)*} = 0 \end{cases} \tag{4}$$

Suppose there are n_{i,r_1,r_2} generated values of $\mathbf{y}$ of which $i^{(1)} = i$ and $i^{(2)} = r_1$ and $i^{(3)} = r_2$ and there are n_{i,r_1} generated values of $\mathbf{y}$ of which $i^{(1)} = i$ and $i^{(2)} = r_1$. Then for company $i\,(1 \le i \le N)$, the probability of the transition from rating r_1 to r_2 is estimated by

$$P_{i,r_1,r_2} = \frac{n_{i,r_1,r_2}}{n_{i,r_1}}, \quad 1 \le r_1, r_2 \le M\ . \tag{5}$$

NUMERICAL RESULTS

Consider the quarterly credit ratings of 10 companies in the electronic sector in 15 years (from 1999 to 2013) taken from the database of the Taiwan Economic Journal. The credit rating ranges from 1 (lowest default risk) to 9 (highest default risk), and 10 represents the default state. Thus, the values of N and M (see Section 1) are both equal to 10 and the vector $\mathbf{y}$ (see Section 1) has 27 components. A multivariate power-normal distribution is first fitted to the 590 observed values of $\mathbf{y}$. Some of the estimated transition probabilities based on the multivariate power-normal distribution for the 10 companies are presented in Table 1 along with those estimated by using the maximum likelihood procedure. Let M_{ij} be the number of times when the rating of company i is j in the current quarter. When M_{ij} is large, the maximum likelihood procedure would be able to give fairly good estimate of the transition probability of the rating of company i in the next quarter given that the rating of company i in the current quarter is j. Table 1 shows that the transition probabilities estimated by the two procedures are fairly comparable when the corresponding M_{ij} is not too small.

TABLE 1. Transition probability estimated by maximum likelihood procedure, and transition probability (in parentheses) based on multivariate power-normal distribution $\left(N_s = 100,000\right)$.

(a) Company 1

Rating This Quarter (*j*)	Rating Next Quarter 7	8	9	M_{1j}
7	0.9773 (0.9089)	0.0227 (0.0102)	0 (0.0200)	44
8	0 (0)	0.8 (0.5369)	0.2 (0.0671)	5
9	0.1 (0.0787)	0 (0)	0.9 (0.794)	10

(b) Company 2

Rating This Quarter (*j*)	Rating Next Quarter 8	9	10	M_{2j}
8	0.8333 (0.6032)	0.1667 (0.0556)	0 (0)	6
9	0.0667 (0.0176)	0.8667 (0.8223)	0.0667 (0.1293)	15
10	0 (0)	0.0263 (0.0579)	0.9737 (0.8542)	38

(c) Company 3

Rating This Quarter (*j*)	Rating Next Quarter 3	4	M_{3j}
3	0.9 (0.8044)	0.1 (0.02235)	10
4	0 (0)	1 (0.946)	49

(d) Company 4

Rating This Quarter (*j*)	Rating Next Quarter 7	8	9	10	M_{4j}
7	1 (0.8737)	0 (0)	0 (0)	0 (0)	24
8	0.5 (0.1056)	0.5 (0.5466)	0 (0)	0 (0)	2

9	0 (0)	0.09091 (0.0192)	0.9091 (0.8237)	0 (0)	11
10	0 (0)	0 (0)	0.0455 (0.05697)	0.9545 (0.8158)	22

(e) Company 5

Rating This Quarter (*j*)	Rating Next Quarter					M_{5j}
	5	6	7	8	9	
5	1 (0.9061)	0 (0)	0 (0)	0 (0)	0 (0)	20
6	0.25 (0.05)	0.75 (0.6333)	0 (0)	0 (0)	0 (0)	4
7	0 (0)	0.5 (0.0035)	0.5 (0.8054)	0 (0)	0 (0)	2
8	0 (0)	0 (0)	0.0833 (0.0578)	0.9167 (0.5632)	0 (0)	12
9	0 (0)	0 (0)	0 (0)	0.0476 (0.0244)	0.9524 (0.801)	21

(f) Company 6

Rating This Quarter (*j*)	Rating Next Quarter			M_{6j}
	6	7	8	
6	1 (0.7672)	0 (0)	0 (0)	14
7	0.0526 (0.0173)	0.8421 (0.8348)	0.1053 (0.04445)	19
8	0 (0)	0.0769 (0.0754)	0.9236 (0.7148)	26

(g) Company 7

Rating This Quarter (*j*)	Rating Next Quarter			M_{7j}
	7	8	9	
7	1 (0.8859)	0 (0)	0 (0)	32
8	0.5 (0.1238)	0.5 (0.5302)	0 (0)	2
9	0 (0)	0.04 (0.0151)	0.96 (0.8358)	25

(h) Company 8

Rating This Quarter (*j*)	Rating Next Quarter			M_{8j}
	8	9	10	
8	0 (0)	1 (0.0829)	0 (0)	1
9	0 (0)	0.9444 (0.8353)	0.0556 (0.1136)	18
10	0 (0)	0.025 (0.072)	0.975 (0.8523)	40

(i) Company 9

Rating This Quarter (*j*)	Rating Next Quarter				M_{9j}
	5	6	7	8	
5	1 (0.9308)	0 (0)	0 (0)	0 (0)	26

6	0.05882 (0.0426)	0.8824 (0.7593)	0.0588 (0.037)	0 (0)	17
7	0 (0)	0.1111 (0.0192)	0.7778 (0.8219)	0.1111 (0.0159)	9
8	0 (0)	0 (0)	0.1429 (0.0584)	0.8571 (0.539)	7

(j) Company 10

Rating This Quarter (*j*)	**Rating Next Quarter** **9**	 **10**	M_{10j}
9	0.8571 (0.8029)	0.1429 (0.1421)	7
10	0.0192 (0.0474)	0.9808 (0.8522)	52

CONCLUSION

The numerical results for the transition probabilities in Section 3 show that the fit given by the multivariate power-normal distribution is fairly satisfactory. A possible reason for the satisfactory fit is that we have used all the companies' rating data to estimate the transition probability matrix of a given individual company. A limitation of the computed transition probabilities is that they are assumed to be time-independent. To obtain time-dependent transition probabilities, we may first fit the data of m ($m < 59$) consecutive quarters with a multivariate power-normal distribution and obtain the transition probabilities for the ratings of the i-th company in the immediate future quarter for $1 \le i \le 10$. In this way we can perform an out-of-sample prediction of the future ratings.

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