Sunway Academic Journal 3, 87–98 (2006)

WAVEFORM DICTIONARIES AS APPLIED TO THE AUSTRALIAN EXCHANGE RATE

SHIRLEY WONG[a]
RAY ANDERSON
Victoria University, Footscray Park Campus, Australia

ABSTRACT

This paper proposes a new method, called waveform dictionaries, to analyze the trend of the exchange rate of the Australian dollar against other currencies. Four different currencies are explored. They are the U.S. dollar, the Japanese yen, the British pound and the euro. The exchange rates can be classified as non-stationary signals. The waveform dictionaries can reduce the complexity of these signals to produce some informative correlation features. These features are sparse in the time-frequency domain and thus can efficiently represent the characteristics of the signal. These features are then displayed in time-frequency (scale) maps. These maps provide possible insights into market behavior such as the dominant market reaction to news.

Key words: Waveform dictionaries, time-frequency map, decompose.

INTRODUCTION

Forecasting is important for business. In spot speculation, the speculator will buy a currency if the forecast shows that it will appreciate, or sell a currency if the forecast shows that it will depreciate. In spot forward speculation, if the spot exchange rate is predicted to be higher than the forward rate on the maturity date of the forward contract, the speculator will buy forward and sell spot upon delivery. In option speculation, a long call or a short put decision will be made if the currency is expected to appreciate. In hedging decisions, the receivables will be hedged if the foreign currency is forecasted to depreciate, and the payables will be hedged if the foreign currency is forecasted to appreciate. Exchange rate forecasting is needed in the case of foreign investments such as setting up a foreign subsidiary. From the macroeconomic perspective, exchange rate forecasting is essential in predicting variables such as inflation. Central-bank intervention in the foreign exchange market involves predicting exchange rate movements.

There are several quantitative approaches to analyzing exchange rates such as the econometric models and time series methods. In single-equation econometric models, prices are available only on a monthly basis and figures on national income are available only on a quarterly basis. Forecasts of the exchange rates cannot be done on a daily basis. These classical data analysis methods are only suitable for stationary signals (time invariant).

Recently, the method of waveform dictionaries has been accepted as a new data analysis tool for non-stationary data and applied in financial markets (Wong et al., 2003). This new data analysis technique is called waveform dictionaries, which are a class of transforms that

E-mail: [a]Shirley.Wong@vu.edu.au.

generalizes both short time Fourier transforms and wavelet transforms (Ramsay and Zhang, 1997). The amount of localization in time and frequency in each dimension is fixed for the former but is automatically adapted for the latter. Each waveform is parameterized by location, frequency, and scale. Such transforms can analyze signals that have highly localized structures in either time or frequency space, as well as broadband structures. Waveforms can, in principle, detect everything from shocks represented by Dirac Delta functions, to short bursts of energy within a narrow band of frequencies that occur sporadically, as well as the presence of frequencies that are held over the entire observed period.

There are many types of wavelet families with different qualities, such as the Daubechies wavelets db2 and db3 (Figure 1). These Daubechies wavelets have been selected because they form an orthogonal basis. Hence, the result of the wavelet transform can be sparse and useful. The selection of the appropriate mother wavelet as a base is probably the most important step in ensuring the accuracy of the data analysis (Daubechies, 1992).

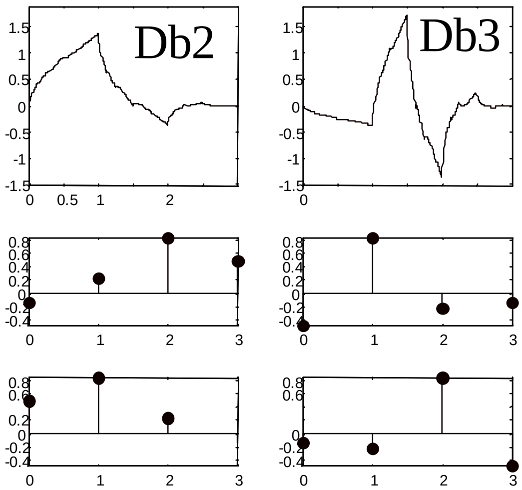

Figure 1. Typical examples of Daubechies wavelets: db2 and db3. A wavelet is like a water ripple that has limited oscillations with amplitudes decaying quickly in milliseconds (local in time, on the *x*-axis) and extracts both time and frequency information locally.

In the next section, we review literature relevant to the paper’s objectives. Then we discuss the concept of waveform dictionaries as applied to the analysis of exchange rates. The results of the waveform analysis are then presented. The final section completes the paper with a summary and conclusion.

LITERATURE REVIEW

Several theories have been developed on the determination of exchange rates. Behind these theories lie two general hypotheses: the Purchasing Power Parity Hypothesis (Bahmani-Oskooee, 1993), and the Uncovered Interest Rate Parity Hypothesis (McCurdy and Morgan, 1991). Under the Purchasing Power Parity Hypothesis, exchange rates will be adjusted in proportion to the changes in national price indices, which means that a currency's real value will be the same at any time. Under the Uncovered Interest Rate Parity Hypothesis, the differences in the interest rates among countries will be equal to the rate of change in the exchange rates.

Many of the studies on exchange rates have been made using classical data analysis techniques, which can be divided into three main approaches. First, the distribution of daily foreign exchange rates can be pattern recognized (Boothe and Glassman, 1987). To test the exchange rates' distribution the following are used: the normal distribution with its mean and standard deviation parameters; the symmetric stable Paretian distribution with the characteristic exponent bounded by zero and two; the Student distribution characterized by its mean, standard deviation and degrees of freedom; and a mixture of two normal distributions with one mean, two standard deviations and a mixing parameter. Second, the statistical values such as means or second moments of data can be used to find the degree of temporal correlation. The Autoregressive Conditional Heteroskedasticity (ARCH) Model has been used to study the relationship between the exchange rates and other economic variables (Bollerslev et al., 1992). Third, the degree of stationarity in the data can be analyzed. Since daily foreign exchange rates have been extended to the new intra-daily data on postings of bid and ask quotes on foreign exchange rates using tick-by-tick observations obtained worldwide, the intra-daily data have not really been accepted to be stationary. Fourth, the waveform dictionaries method is a new data analysis tool for non-stationary data and has been applied in foreign exchange rate studies (Wong et al., 2003).

FROM FOURIER ANALYSIS TO WAVELET ANALYSIS

The Fourier transform decomposes a function into simple sine periodic functions as basis functions without time information. The equation:

(1) $$x(t) = \sum_k C_k e^{jk\omega t}$$

is valid for any periodic function. The frequency parameter is ω and (1) represents a superposition of harmonics of sine functions in terms of multiples of ω. The coefficients are given by the integral:

(2) $$C_k = \frac{1}{2\pi} \int_0^{2\pi} x(t) e^{-jk\omega t} dt$$

Each coefficient can be seen as the average harmonic content of $x(t)$ at frequency ω. Thus, the Fourier transform can give frequency components of signals without space or time information.

While this approach leads to good results in many applications, some inherent weaknesses are evident. The complete loss of time or space information leads to an insufficient description of a discontinuity or a localized high-frequency spike. The underlying reason causing this effect is the nature of complex exponential functions used as basis functions. All exponential functions cover the entire real line, and differ only with respect to frequency. Thus, a better representation of both the time and frequency domain is needed.

There are two basic approaches to time-frequency analysis. The first approach is to initially cut the time series into fixed time segments and then analyze each of these segments separately to review their frequency content. The other approach is to initially transform the time series into different frequency bands, so that each of these bands can be cut into fixed time segments and then analyzed for their energy content. The first of these approaches is used as the basic principle of the short time Fourier transform, while the second approach, the wavelet transform, is the main focus of this paper.

The short time Fourier transform can analyze a spectrum by cutting up a time series into fixed time segments and then applying Fourier analysis to each segment of the non-stationary signal. However, the constant time-frequency localization in each dimension limits the analysis of many local frequencies or regularities. The auto-adapted windowed time-frequency localization approach is perhaps the most advantageous since the wavelet performs better than the short time Fourier transform (Figure 2).

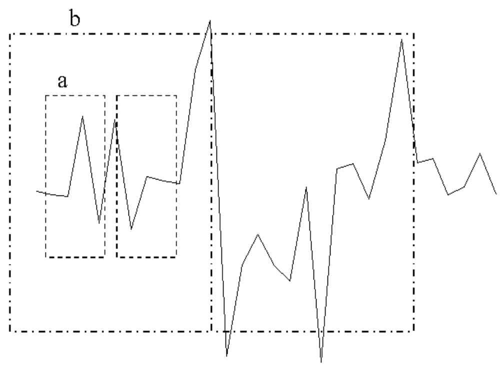

Figure 2. Adaptive short time segments for a vibration transient: Using wavelet analysis, small segments localize high-frequency spikes (a), while large segments provide temporal information for low-frequency trends (b). The *x*-axis and *y*-axis denote the time and magnitude, respectively.

The wavelet transform of a signal *f(x)* depends on two variables: scale (or frequency) parameter *a*, and time (or position) parameter *b*. The wavelet coefficient is obtained by integrating the product of the wavelet with the signal:

(3) $$(W_\Psi f)(a,b) = \langle f, \Psi_{a,b} \rangle = \int \frac{1}{\sqrt{|a|}} \Psi\left(\frac{x-b}{a}\right) f(x)dx$$

$W_\Psi f$ is the wavelet transform of *f* with analyzing wavelet Ψ as the basis function. $\Psi_{a,b}(x)$ denotes that the wavelet Ψ is dilated by *a* and translated by *b*. The parameters *a* and *b* are real numbers (with the constraint $a \neq 0$).

For effective applications, the discrete dyadic wavelet transform with $a = 2^m$ and $b = n2^m$ (*m* and *n* being integers) is applied. Thus:

(4) $$Wf[m,n] = \langle f, \Psi_{m,n} \rangle = \int_R f(x)\Psi_{m,n}(x)dx$$

(5) $$\Psi_{m,n}(x) = 2^{-m/2}\Psi(2^{-m}x - n)$$

When scale *m* decreases, the time support is reduced but the wavelet Heisenberg box shifts towards high frequencies. The Heisenberg box is a 2-dimensional block in a time-frequency map. When *m* increases, the frequency support of the wavelet is shifted towards low frequencies. The time resolution increases while the frequency resolution decreases.

WAVEFORM DICTIONARIES IN THE ANALYSIS OF EXCHANGE RATES

Waveform dictionaries are a class of transforms that generalizes both short time Fourier transforms and wavelets. The former consists of segmenting the signal into windows of fixed lengths and represents functions that are highly localized in frequency space, while the latter represents signals that are tightly localized in time. The application of the wavelet transforms to the exchange rate signal is shown in Figure 3.

The technique that applies wavelets to exchange rates has the properties of locality, multi-resolution and compression that are suitable for analyzing non-stationary data such as exchange rates. Each wavelet is localized in time and frequency domains concurrently. A narrow time-window is used to investigate the high-frequency components of a signal while a wide time-window is used to examine the low-frequency components of the signal. Wavelet atoms are compressed and dilated to analyze at a flexible resolution using varying levels of focus. The wavelet transforms of signals can be sparse so that the wavelet coefficients can be small.

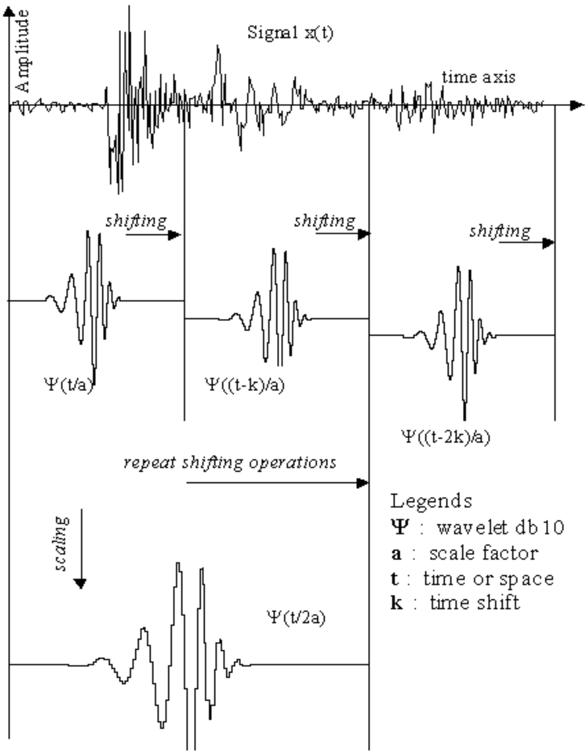

Figure 3. The exchange rate signal is decomposed by the wavelet transform (WT) into shifted and scaled versions of the mother wavelet $\Psi(t)$, Daubechies wavelet db10. The WT acts as a sort of mathematical microscope through which segments of the signal may be examined by adjusting the scale.

The breakdown of the exchange rate signal into segments is the key to the fast algorithm. Given a signal *x* of length *N*, the formation of the discrete wavelet transform consists of $\log_2 N$ stages at the most. First, a decomposition separates the signal into low-frequency and high-frequency segments using x = a + d, with a representing the smooth coefficients and d the unsmooth, remaining or difference coefficients of *x*. The coefficients are obtained through the discrete convolution of *x* with a low-pass filter for the smooth coefficients, and a high-pass filter for the difference coefficients. A dyadic decimation (down-sampling) follows. Further splitting of the smooth part of the signal in analog form gives a multiscale analysis of the signal. Using this approach, the wavelet features of the signal can be parameterized and displayed in a 2-D Cartesian plane, namely a time-frequency map, also known as an energy map (Figure 4).

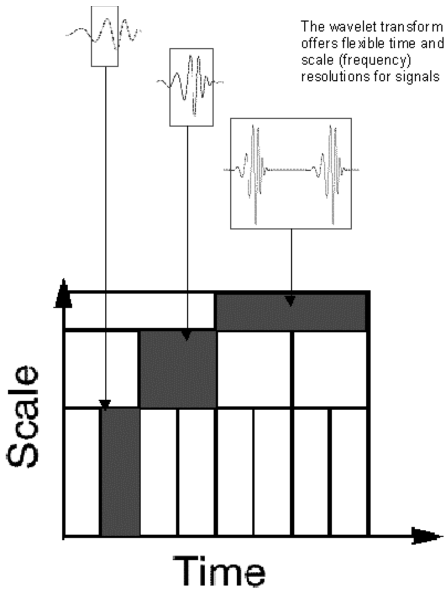

Figure 4. Illustration of flexible time-scale (frequency) resolutions obtained through a wavelet analysis. Scale increases from the bottom to the top of the *y*-axis and frequency is inversely proportional to scale.

RESULTS OF WAVEFORM DICTIONARIES

We attempted to gain insights by using waveform dictionaries to analyze foreign exchange rates. The specific data that we used were sampled from daily observations collected by the University of British Columbia. The raw data dates from 1/2/1998 to 15/4/1999. The four exchange rates examined were the U.S. dollar–Australian dollar, the Japanese yen–Australian dollar, the British pound–Australian dollar, and the euro–Australian dollar. The daily sampling rate was deemed appropriate since it avoided the periodicity that was induced by the institutional structure of the international market for exchange rates in major world currencies.

The wavelet transform decomposes the four different signals into separate time-frequency (scale) maps (see Figures 5 to 8). Figure 5 shows the Japanese yen–Australian dollar exchange rate. The time-frequency map indicates the highest energy location (in deepest gray), that is, 153 working days from the starting date, 1/2/1998. The detail d_1 component detects a high exchange rate spike on that day. The detail d_1 shows the high-frequency activities of the exchange rate market while the low-frequency a_1 approximates the trend of the market. Also, the maximum energy bin is found at scale 1 (level 1) of the *y*-axis. This indicates that the signal's nature is smooth and regular. Similar analyses can be applied to the other figures (see Figures 6 to 8).

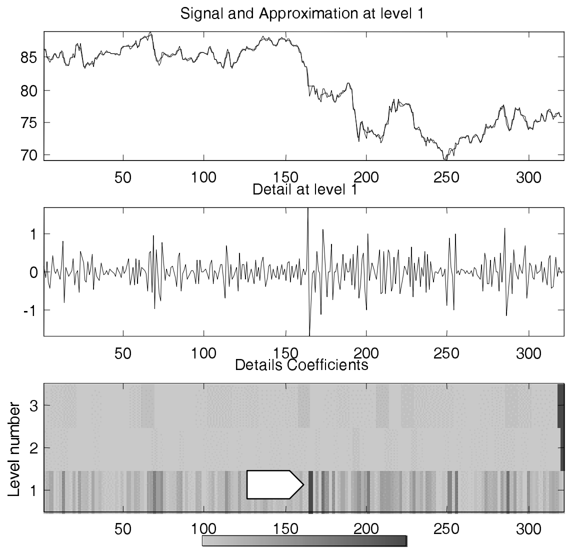

Figure 5. Japanese yen/A$ in 1998. The gray-scale map (bottom) shows the time and frequency localization of the exchange rate: the *x*-axis shows the working date starting from 1/2/1998; the *y*-axis shows the frequency (scale) resolution. The deepest gray location on the map shows the location with the highest energy.

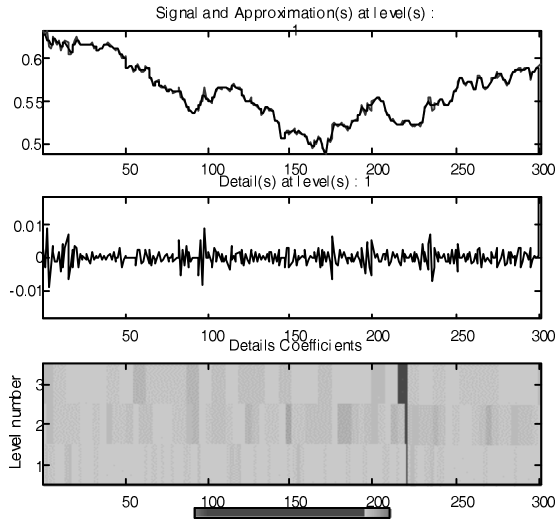

Figure 6. Euro/A$ in 1998

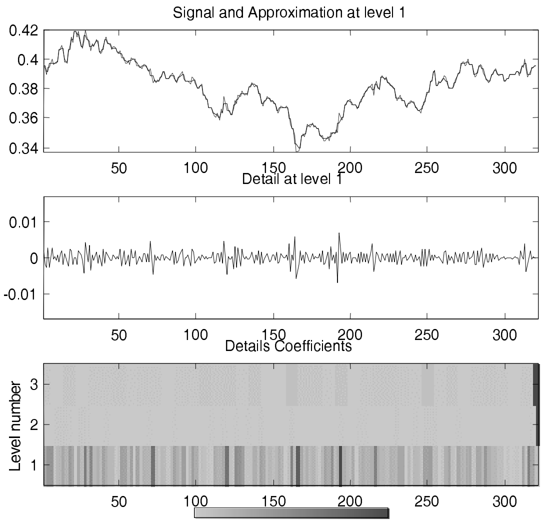

Figure 7. GBP/A$ in 1998

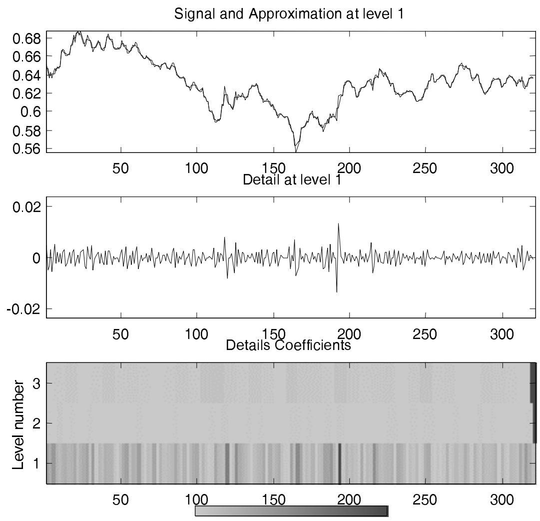

Figure 8. US$/A$ in 1998

The wavelet decomposition features at details d_1 and d_2, and the approximate a_2 for the US$/A$ exchange rate is shown in Figure 9. The signal s is compared to a_2 that indicates the trend of the dollar. The detail d_1 shows the high-frequency market activities at the sampling rate of 1 datum per 2 days, while d_2 shows the lower-frequency market activities at half of the frequency of d_1, that is, 1 datum per 4 days. The curve d_2 shows that the low-frequency cycle is approximately 20 working days.

The cumulative frequency of the signal US$/A$ is shown in Figure 10. The highest activity moving the price level is found at US$0.623 within the inter-quartile range.

Histograms of the low-frequency approximate a_1 and the high-frequency detail d_1 of US$/A$ are shown in Figures 11 and 12.

The overall results indicate that the waveform dictionaries can work as filters that can cut the exchange rate into smaller bands of frequencies: scale levels 1 and 2 shown in Figures 9 to 12.

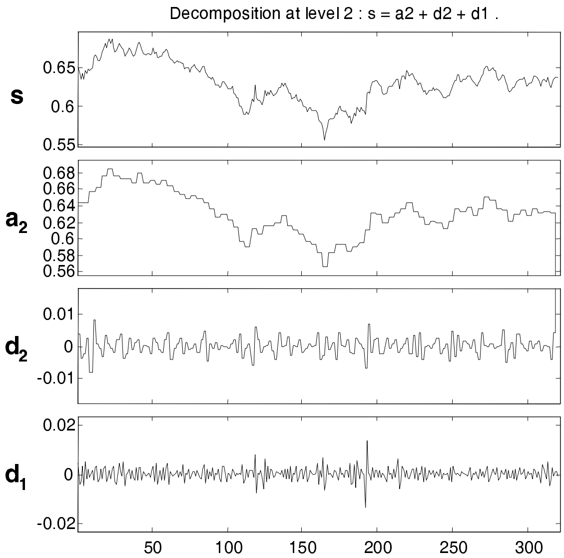

Figure 9. The wavelet transform decomposes the US$/A$ exchange rate to the wavelet coefficients approximate a_2 and details d_1 and d_2

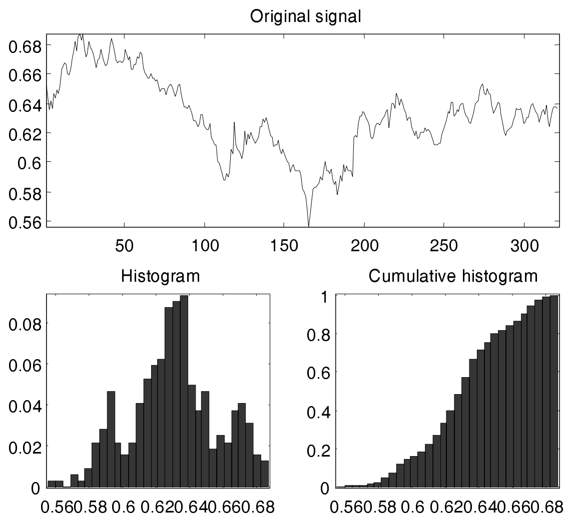

Figure 10. Cumulative frequency of the signal US$/A$ in 1998. The *x* and *y* axes denote price level and activity, respectively.

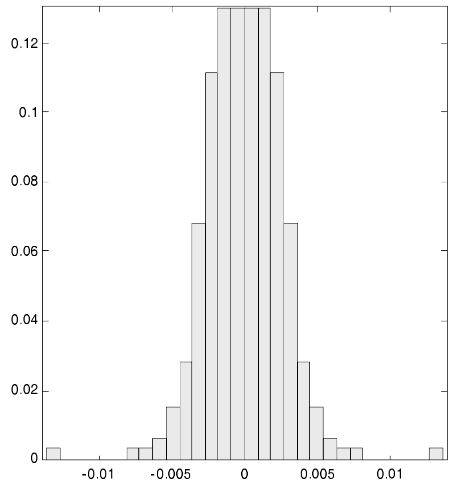

Figure 11. Histogram of the approximate a_1
of the signal US$/A$ in 1998

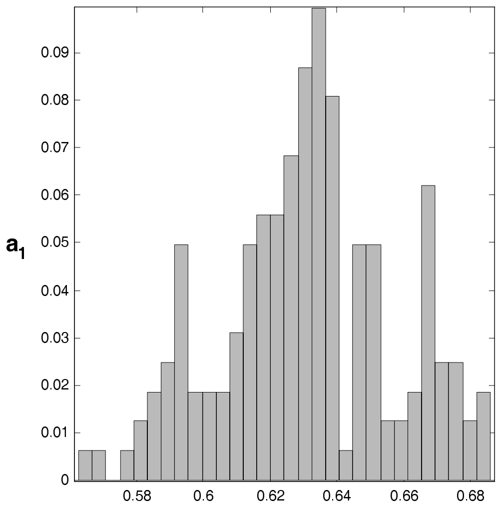

Figure12. Histogram showing a high-
frequency d_1 of the signal US$/A$ in 1998

Each band provides a more detailed representation of a small portion of the signal. This approach provides an alternative way to extract useful information regarding expectational changes in variables such as interest rates, inflation, trade balances and money supply from spikes or transients that may not be sustained throughout the entire period of observation of the data contaminated by noise. Furthermore, the transients or spikes of the data may represent short-run bursts of market activity or energy over a narrow range of contiguous frequencies. These localized frequency bursts not only represent most of the signal's energy but also give significant insights into market behavior. These bursts may be viewed as the dominant market's reaction to news. They can be observed clearly at the first difference or detail d_1: Dirac Delta functions are found. The median of changes is nearly invariant at zero for d_1. This indicates an almost even chance that the price will rise or fall.

The advantage of this approach has become clear by studying a limited amount of data in this paper. To improve forecasting potential, more data need to be collected and experiments carried out. This approach represents a significant improvement in financial market forecasting.

CONCLUSION

The proposed waveform dictionaries approach has been shown to provide new information on exchange rates compared to the traditional Fourier-based methods, which are incapable of dealing with a signal that is changing over time. The data used in the analysis was sampled from daily observations. Four different exchange rates were analyzed and the results are shown in the time-frequency maps. These maps provide possible insights into market behavior such as the dominant market reaction to news.

REFERENCES

Bahmani-Oskooee, M. (1993). Purchasing power parity based on effective exchange rate and cointegration: 25 LDCs' experience with its absolute formulation. *World Development*, 21, 1023–1031.

Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance: a review of the theory and empirical evidence. *Journal of Econometrics*, 52, 5–59.

Boothe, P., & Glassman, D. (1987). The statistical distribution of exchange rates: empirical evidence and economic implications. *Journal of International Economics*, 22, 297–319.

Daubechies, I. (1992). *Ten lectures on wavelets*. Philadelphia, PA: SIAM.

McCurdy, T. H., & Morgan, I. G. (1991). Tests for a systematic risk component in deviations from uncovered interest rate parity. *The Review of Economic Studies*, 58(3), 587–602.

Ramsay, J. B., & Zhang, Z. (1997). The analysis of foreign exchange data using waveform dictionaries. *Journal of Empirical Finance*, 4(4), 341–372.

Wong, H., Ip, W. C., Xie, Z. J., & Lui Xue Li (2003). Modeling and forecasting by wavelets, and the application to exchange rates. *Journal of Applied Statistics*, 30(5), 537–553.