## The Prediction of Moment of Inertia of Rotating Nuclei\*

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**Abstract:** In this paper the mathematical expression which is given by Bohr for the moment of inertia of even - even nuclei on the basis of the hydrodynamical model is modified. This modification is on the kinetic energy of the surface oscillations including the second and third terms of *R*- expansion as well as the first term which was already carried out by Bohr. Therefore, this work can be considered the continuation and support of the hydrodynamic model of Bohr. This procedure results in a Bohr formula to be multiplied by a factor which depends on the deformation parameter. Bohr (modified) formula is examined by applying it on axially symmetric even-even nuclei with atomic mass ranged between 150 and 190 as well as to some triaxial symmetry nuclei. The modification of Bohr's formula are discussed including the information on how stable this modification with including second and third terms of *R*- expansion of Bohr's formula. The results of calculation are compared with the experimental data and the results of Bohr, based earlier. The obtained results are in a good agreement with experimental data by describing almost 0.7 and better than that of the unmodified ones.

**Keywords:** Moment of inertia, Hydrodynamical model, Irrotational motion **DOI:** 

## 1 Introduction

The problem of calculating the moment of inertia (MOI) of even-even nuclei has received a considerable attention since 1950's Refs. [1-3]. The hydrodynamical model, firstly introduced by Bohr [4], considered the nucleus as a droplet of incompressible irrotational fluid. As a consequence, the collective motion of the nucleus was pictured as a quadrupole classical oscillations similar to the oscillations of the liquid droplet that had been discussed in detail by Rayleigh [5]. This description of Bohr leads to simple relationship between moment of inertia 3 and deformation parameter  $\beta$ , as  $\mathfrak{I} = 3B\beta^2$ , where B is the inertial parameter and this relationship is called Bohr's formula. In particular case of small oscillations and hence  $R(\theta, \phi)$ , the radial coordinate of the surface at polar coordinates  $(\theta, \phi)$  can be approximated to  $R_0$ , the radius of the equilibrium spherical shape of the liquid drop, Rayleigh's calculations showed that  $B = \frac{\rho R_0^5}{2}$ . The values of the *MOI* calculated using Bohr's formula with the values of *B* derived by Rayleigh, are five times smaller than from the values of *MOI* which is obtained experimentally. Furthermore, one cannot explain the *MOI* of deformed nuclei by considering the extreme case of a rigid deformed shape [6].

An alternative approach to describe the MOI of deformed nuclei was based on the cranking model which was introduced by Inglis [7]. In this model the kinetic energy of rotation is obtained by considering the motion of the nucleons in rotating self-consistent field. In contrast to the hydrodynamical model, the results of the cranking model are found to be 2-3 times larger than that of the experimental ones. Since then, both models: the hydrodynamical model of Bohr and the cranking model of Inglis have been modified by several authors Refs. [2, 8-14]. Recently, a valuable thorough quantitative comparison of predictions for both form factor and moment of inertia of four different models (Hartree-Fock, cranking model, rigid rotator and irrotational fluid flow) for the rare earth nuclei  $^{154}Sm$ ,  $^{156}Gd$ ,  $^{164}Dy$ ,  $^{166,168}Er$  and  $^{174}Yb$  were proposed in reference [9]. The authors of reference [9] extended their work to observed the electromagnetic form

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