# SOVA decoding in Symmetric Alpha-Stable Noise

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***Abstract*— Soft-Output Viterbi Algorithm (SOVA) is one type of recovery memory-less Markov Chain and is used widely to decode convolutional codes. Fundamentally, conventional SOVA is designed on the basis of Maximum A-Posteriori Probability (APP) with the assumption of normal distribution. Therefore, conventional SOVA fails miserably in the presence of symmetric alpha stable noise $S\alpha S$ which is one form of stable random processes widely accepted for impulsive noise modeling. The author studies and has improved the performance of conventional SOVA by introducing Cauchy function into path-metric calculation. Substantial performance improvement was gained from Mento Carlo Simulation for SOVA based turbo codes.**

***Keywords-component:* *turbo codes,* Soft Output Viterbi Algorithm (*SOVA), Maximum A-Posteriori (MAP) Estimation, Cauchy metric, Symmetric Alpha Stable* ($S\alpha S$ *) Distribution, non-gaussian CHANNEL.***

## I. INTRODUCTION

Power line communication (PLC) for data transmission has been the fascinating topic for coding theorists, scientists and engineers for broadband communications since 20th century. However, the data that is transmitted over such channels would be subjected to various interferences; such as impulsive noise and Additive White Gaussian Noise (AWGN). Conventional channel codes that are typically optimum in the context of AWGN impairment are found deficient in the presence of impulsive noise. Several models have been developed to statistically describe the behaviors of non-Gaussian channels. They are contaminated Gaussian model, Generalized Gaussian Distribution(GCD), Stable Distributions and Middleton's Class. Among the interference model, Symmetric Alpha-Stable model $S\alpha S$ was chosen to emulate the noisy environment of the transmission medium for PLC in our investigation.

Iterative decoding of turbo codes has received a lot attentions from researchers since its invention and has been proven to approach Shannon capacity limit with minimum required $E_b/N_0$ of 0.7 dB for BER of $10^{-5}$ [1, 2,3].

Typically, turbo codes can be categorized according to its concatenated connections, which are parallel concatenated convolutional codes (PCCC) and serial concatenated convolutional codes (SCCC). The component codes of PCCC and SCCC are connected parallel and serial respectively [3, 4]. PCCC was used in our analysis to serve as the platform for SOVA based turbo codes. The encoder of PCCC is illustrated as Fig. 1:

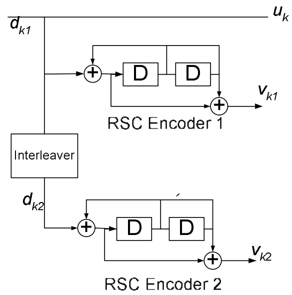


Figure. 1 PCCC encoder with code rate $r = 1/3$.

The encoder of parallel turbo codes consists of two parallel recursive convolutional component encoders with the random interleaver to minimize the symbol correlation between two inputs. In order to increase code rate, puncturing can be performed on parity bits from first and second component encoders alternatively. Subsequently, multiplexer can be used to produce desired outputs. The code rate can be defined as Eqn. 1.

$$R = \frac{k}{n} \tag{1}$$

where *k* is the information bits and *n* is the output bits .

In the presence of non-Gaussian noise, the performance of turbo codes is substantially degraded. It is possible to mitigate the detrimental effects brought into turbo codes by impulsive noise. It is shown in [5] that the introduction of Cauchy probability function into the detection of MAP (Maximum A-Posteriori) decoder has enabled the conventional turbo codes to function effectively over non-Gaussian channel and significant improvement in BER was obtained. The performance improvement was achieved in previous work by introducing robust Cauchy probability density function (pdf) into MAP's transition metric computation. However, MAP decoder is highly complicated for physical VLSI applications and incurs intolerable decoding latency for time critical communication systems. Hence, a low complexity decoding strategy was proposed in [6] by enabling the widespread use Viterbi algorithm (VA) with soft-decision output capability and it is known as Soft-Output Viterbi Algorithm (SOVA). In our investigation, the author has further extended the conventional AWGN optimized SOVA to provide optimum performance in Symmetric Alpha Stable ( $S\alpha S$ ) channel.

To decode the parallel concatenated convolutional codes which are produced by parallel convolutional code, SOVA decoder is used and its block diagram is shown in Fig. 2:

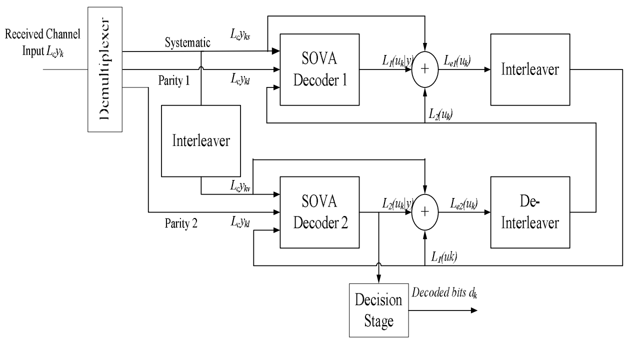


Figure. 2 SOVA PCCC decoder.

The received information would be decoded iteratively with SOVA component decoders. Extrinsic information is produced by subtracting soft-decision output $L(\mathbf{u}\,|\,\mathbf{y})$ with a-prior information $P(\mathbf{u})$ and channel measurement via matched filter. The extrinsic information represents the addition uncorrelated gain from turbo decoding. The soft-output extrinsic information $L_e(\mathbf{u})$ would be exchanged between the component decoders and better estimate for particular output bit is obtained for additional iteration of turbo codes. The gain of the turbo codes is its iterative nature of passing extrinsic information to aid subsequent stage of decoding performance resembling to turbo engine.

Typically, the BER of the decoded bits will fall exponentially for additional iteration that is performed on the pairs of SOVA component decoders. For each iteration, the performance improvement is also decrease exponentially. For reasonable complexity and decoding latency, eight iterations are used as additional iteration has shown insignificant and little performance improvement over decoding bits[14].

## *II. CHANNEL AND NOISE MODELS*

To generate impulsive noise for simulation purpose, Symmetric Alpha Stable ( $S\alpha S$ ) model is chosen in our investigation due to its excellent empirical fits on data and many signal processing applications are symmetric. Typically, $S\alpha S$ distribution is characterized by setting its skewness parameter $\delta$ to zero. Its characteristic function is given in Eqn. 2.

$$\phi(\omega) = e^{-\gamma^{\alpha}|\omega|^{\alpha}}, \quad -\infty < \omega < \infty \qquad (2)$$

where $\gamma$ is dispersion and $\alpha \in (0, 2]$ is the characteristic exponent which described the impulsiveness of $S\alpha S$ process. When $\alpha = 2$ , it gives Gaussian distribution and when $\alpha = 1$, Cauchy distribution could be obtained from the random process. Due to the non-close form for other values of $\alpha$ , our investigation is limited to $\alpha = 1$ to $\alpha = 2$.

To convenient our derivation for robust SOVA, the variables for our transmission model are defined as vector *u* is denoted as the input vector $u = [u_1, u_2, u_3, ......u_n]$ . Vector *y* is the received vector $y = [y_1, y_2, y_3, .........y_n]$ from the noisy channel. Vector *x* is transmitted sequence produced by the pair of parallel convolutional encoders $x = [x_1, x_2, x_3, ........x_n]$ .

The baseband received signal prior to matched filtering could be described mathematically as Eqn. 3:

$$\mathbf{y} = A\sqrt{E_b}\,(2 \cdot \mathbf{x} - 1) + \mathbf{n} \qquad (3)$$

where *A* is the channel gain and *n* is the random process with normal or $S\alpha S$ random variables.

## *III. ROBUST SOVA DECODER*

Theoretically, SOVA is derived based on MAP by extending the functionality of Viterbi algorithm (VA) to provide soft–decision output. MAP can be expressed as mathematically as Eqn. 4 which maximizes the a-posteriori probability (APP) of the decoded bits:

$$\hat{u} = \arg\max\{p(\mathbf{u}|\mathbf{y})\}$$
$$= \arg\max\{p(\mathbf{y}|\mathbf{u})\,p(\mathbf{u})\} \tag{4}$$

The soft-decision output of SOVA is given as a-posteriori (APP) Log-Likelihood Ratio (LLR ) as Eqn. 5:

$$L(u_k \mid \boldsymbol{y}) = \ln\left(\frac{P(u_k = +1 \mid \boldsymbol{y})}{P(u_k = -1 \mid \boldsymbol{y})}\right) \tag{5}$$

Eqn. 5 can be computed recursively by incorporating trellis as follow [8]:

$$L(u_k \mid \mathbf{y}) = \ln\left(\frac{\sum_{u_k=+1} P(s_{k-1}, s_k, \mathbf{y})}{\sum_{u_k=-1} P(s_{k-1}, s_k, \mathbf{y})}\right)$$
$$L(u_k \mid \mathbf{y}) = \ln\left(\frac{\sum_{u_k=+1} \alpha_{k-1}(s_{k-1})\cdot\gamma_k(s_{k-1}, s_k)\cdot\beta_k(s_k)}{\sum_{u_k=-1} \alpha_{k-1}(s_{k-1})\cdot\gamma_k(s_{k-1}, s_k)\cdot\beta_k(s_k)}\right) \tag{7}$$

Where $u_k = +1$ is the set of input to the SOVA decoder that resulted in the transition from previous state $s_{k-1}$ to the present state $s_k$ and hence similar to $u_k = -1$ $\alpha_{k-1}(s_{k-1})$ is the forward recursion of APP. $\gamma_k(s_{k-1}, s_k)$ is the transition APP from state $s_{k-1}$ to $s_k$ and $\beta_k(s_k)$ is backward recursion of APP.

To reduce the complexity of calculation, the APP-LLR from Eqn. 7 can be computed recursively in natural logarithmic domain as Eqn. 8:

$$L(u_k \mid \boldsymbol{y}) = \ln\left(\frac{\sum_{\substack{(`s,s)\Rightarrow \\ u_k=+1}} \exp(\Gamma_k(`s,s) + A_{k-1}(`s) + B_k(s))}{\sum_{\substack{(`s,s)\Rightarrow \\ u_k=-1}} \exp(\Gamma_k(`s,s) + A_{k-1}(`s) + B_k(s))}\right) \tag{8}$$

where

$$A_k(s_k) = \ln(\alpha_{k-1}(s_{k-1})\cdot\gamma_k(s_{k-1}, s_k))$$
$$B_{k+1}(s_{k+1}) = \ln(\beta_k(s_k)\cdot\gamma_k(s_{k-1}, s_k))$$
$$\Gamma_k(s_{k-1}, s_k) = \ln(\gamma_k(s_{k-1}, s_k))$$

Cauchy metric is obtained from Cauchy distribution as Eqn. 9:

$$f(y) = \frac{1}{\pi}\frac{\gamma}{\gamma^2 + (y-x)^2} \tag{9}$$

with Cauchy Density Function ($\gamma$, $\beta$), where $\gamma$ is the dispersion parameter and it relates to variance as $\sigma^2 = 2\gamma$. Hence the conditional probability of the received symbol can be expressed as

$$p(\mathbf{y_k} \mid \mathbf{x_k}) = \prod_{l=1}^{N} \frac{1}{2\pi}\cdot\frac{\gamma}{\left[\gamma^2 + (y_{kl} - a\cdot x_{kl})^2\right]^{\frac{3}{2}}} \tag{10}$$

where *a* is the fading amplitude of the channel where *a = 1* for non-fading AWGN channel. To enable the effective decoding of SOVA in impulsive noise, modification on the branch metric of conventional Gaussian-based SOVA is needed for robust detection. To equip the Gaussian-based SOVA decoder for robustness, the path-metric could be modified as Eqn. 11:

$$M(s_k) = \ln(p(s_{k-1}, y_{j\le k-1})) + \ln(p(y_k, s_k \mid s_{k-1}))$$
$$= M(s_{k-1}) + \ln(p(y_k \mid s_k, s_{k-1})) + \ln[p(s_k \mid s_{k-1})] \tag{11}$$

where $p(s_k \mid s_{k-1}) = p(u_k)$ is the priori probability of the input bit $u_k = \pm 1$ and $p(y_k \mid s_{k-1}, s_k) = p(y_k \mid x_k)$. Hence, the accumulated path-metric $M(s_k^s)$ that can be computed recursively as Eqn. 12:

$$M(s) = C_1 + M(s_{k-1}) + \frac{u_k}{2}\cdot L(u_k)$$
$$- \frac{3}{2}\sum_{l=1}^{N} \ln\left(\left(\gamma^2 + (y_{kl} - a x_{kl})^2\right)\right) \tag{12}$$

where constant $C_1 = N\ln\left(\frac{\gamma}{2\pi}\right)$ can be omitted. If two paths merge at state $s_k$, then path $s_k$ is selected on the basis that $M(s_k) > M\left(\hat{s}_k\right)$. Then the path metric difference $\Delta_k^s$ of the merging path at stage *k* can be determined which is the magnitude of the soft-decision output of SOVA as the Eqn. 13:

$$\Delta_k^s = M(s_k) - M\left(\hat{s}_k\right) \ge 0 \tag{13}$$

Hence, the soft-decision output of SOVA can be expressed as Eqn. 14 for optimal performance.

$$L(u_k \mid y_k) = u_k \cdot \min_{\substack{j=k\ldots n \\ u_k \neq \hat{u}_k^{sk}}} \Delta_k^s \tag{14}$$

The extrinsic information $L_e(\mathbf{u})$ could be obtained mathematically as Eqn. 15.

$$L_e(u_k) = L(u_k \mid y) - L(u_k) - L_{sys}(u_k) \tag{15}$$

where $L_{sys}(u_k) = \frac{3}{2}\left[\ln(C_2 + 2\cdot y_{ks}) - \ln(C_2 - 2\cdot y_{ks})\right]$ and $C_2 = \gamma^2 + {y_{ks}}^2 + 1$. $L(u_k)$ is the priori Log Likelihood Ration (LLR) from information bits and $L_{sys}(u_k)$ is the LLR from systematic bits of the received signal. Finally, the hard decoded bits can be obtained mathematically from second SOVA component decoder LLRs output as Eqn. 15:

$$\hat{u}_k = \frac{\mathrm{sgn}\left(L\left(u_k \mid y_k\right)\right)+1}{2} \tag{15}$$

where $\mathrm{sgn}\left(x\right)$ is a signum function.

## IV. RESULT AND DISCUSSION

Mento Carlo simulation was performed on the modified robust SOVA PCCC over impulsive $S\alpha S$ channel. Numerical results for BER performance with respect to $E_b/N_0$ were collected and analyzed. In our simulation, binary information bits with 1000 bits and 10 frames were channel coded with parallel convolutional encoder of code rate $r = 1/3$ and be transmitted directly as baseband signal over noisy transmission medium. Random interleaver's size is set to 1000. At receiver, the baseband signals which were corrupted by AWGN or $S\alpha S$ noise were decoded iteratively via SOVA algorithm.

*1)* Performance in AWGN Noise with Bayesian Gaussian metric.

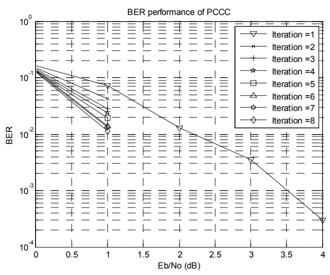


Figure 3. BER performance for SOVA with Bayesian Gaussian metric over AWGN channel.

From the Figure 3, it is shown that SOVA PCCC can perform close to Shannon's capacity limit in AWGN noise.

*2)* Performance in $S\alpha S$ Noise with Bayesian Gaussian metric.

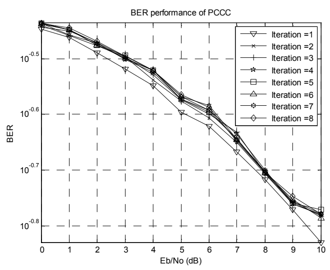


Figure 4. BER performance for SOVA with Bayesian Gaussian metric over $S\alpha S$ channel with , $\alpha = 1$.

However, significant performance degradation can be observed from Figure 4 while conventional SOVA attempted to correct errors due to $S\alpha S$ noise with $\alpha = 1$.

*3)* Performance in $S\alpha S$ Noise with Bayesian Cauchy metric

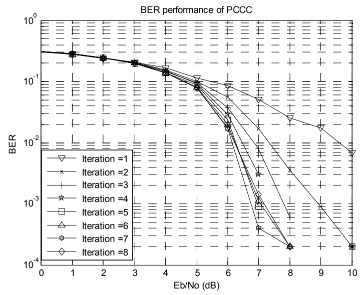


Figure 5. BER performance for SOVA with Bayesian Cauchy Metric over $S\alpha S$ channel with $\alpha = 1$.

Figure 5 shows the performance of modified SOVA with Bayesian Cauchy metric. Performance improvement can be observed from the graph after eight iterations were performed on the received data.

## V. REFERENCE

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